# Sixth Annual Upper Peninsula
# High School Math Challenge

## Northern Michigan University (Marquette Co, MI)

Saturday 14 March 2015

NAME: _SOLUTION_

TEAM: _____

SCHOOL: _____

**PROBLEM 1**                                    **TIME: 3 minutes**

19

_____

**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Let $f(x) = 3x + 1$ and $g(x) = 2x - 4$

Find $(f \circ g)(5)$.

$$(f \circ g)(5) = f(g(5))$$
$$= f(2 \cdot 5 - 4)$$
$$= f(6)$$
$$= 3 \cdot 6 + 1$$
$$= 19$$

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**PROBLEM 2**                                    **TIME:  3 minutes**

90

_____

**answer**

**Put no work on this side of the paper.  Write the answer and only the answer in the space above.  Put all work on the other side of the sheet.**

A positive integer is called a *palindrome* if it reads the same forward as it does backward. For example, 959 and 8228 are palindromes, whereas 1332 is not. Neither the first nor the last digit of a palindrome can be 0.

How many three-digit palindromes are there?

$$a\,b\,a$$

$a$ can be any digit except zero (9 possibilities)

$b$ can be any digit (10 possibilities)

$$9 \cdot 10 = 90$$

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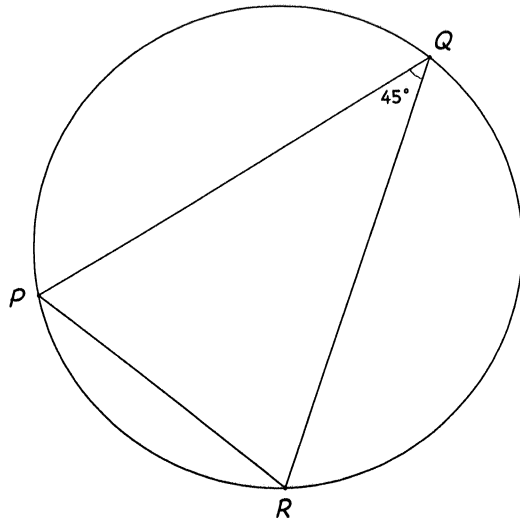NAME: _SOLUTION_

TEAM: _____

SCHOOL: _____

**PROBLEM 3**                                    **TIME: 4 minutes**
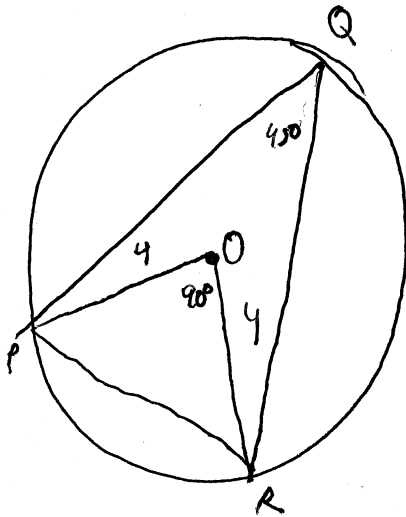
$4\sqrt{2}$

_____

**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Points $P$, $Q$, and $R$ lie on a circle of radius 4 cm.  The measure of $\angle PQR$ is 45°.  Find the length of chord $PR$.



$\angle Q$ subtends an arc of 90°

So, $\angle O$ (O is the center) must be 90°

The hypotenuse of an isosceles right triangle is $s\sqrt{2}$

So, $PR = 4\sqrt{2}$

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**PROBLEM 4**                                                    **TIME: 3 minutes**

436

**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

The ratio of men to women enrolled in a class is 5:4. If the total number of students enrolled in the course is 981, how many women are enrolled?

$m =$ # of men

$w =$ # of women

$$\frac{m}{w} = \frac{5}{4}$$

$m + w = 981$

$4m = 5w$

$4m - 5w = 0$

$-4m - 4w = -3924$

$-4m - 4w = -3924$

_____

$-9w = -3924$

$w = 436$

NAME: _SOLUTION_

TEAM: _____

SCHOOL: _____

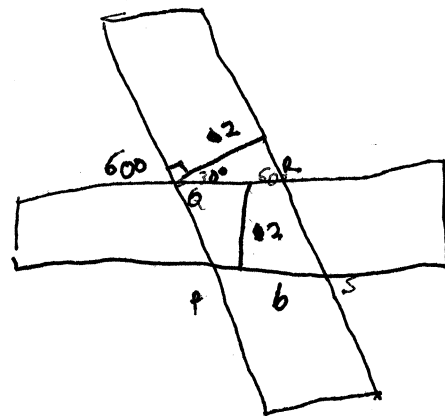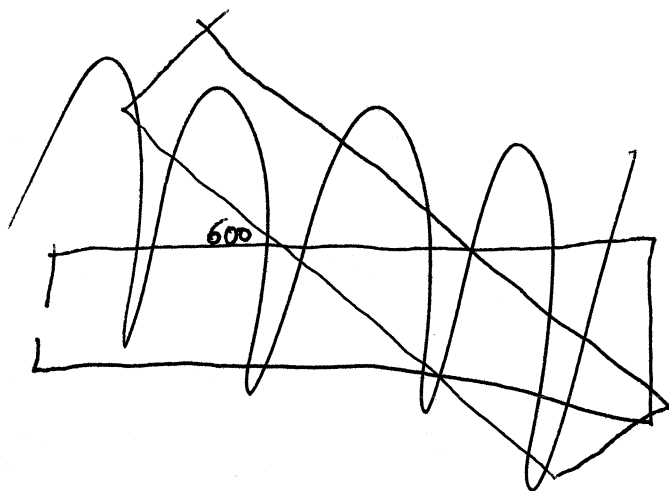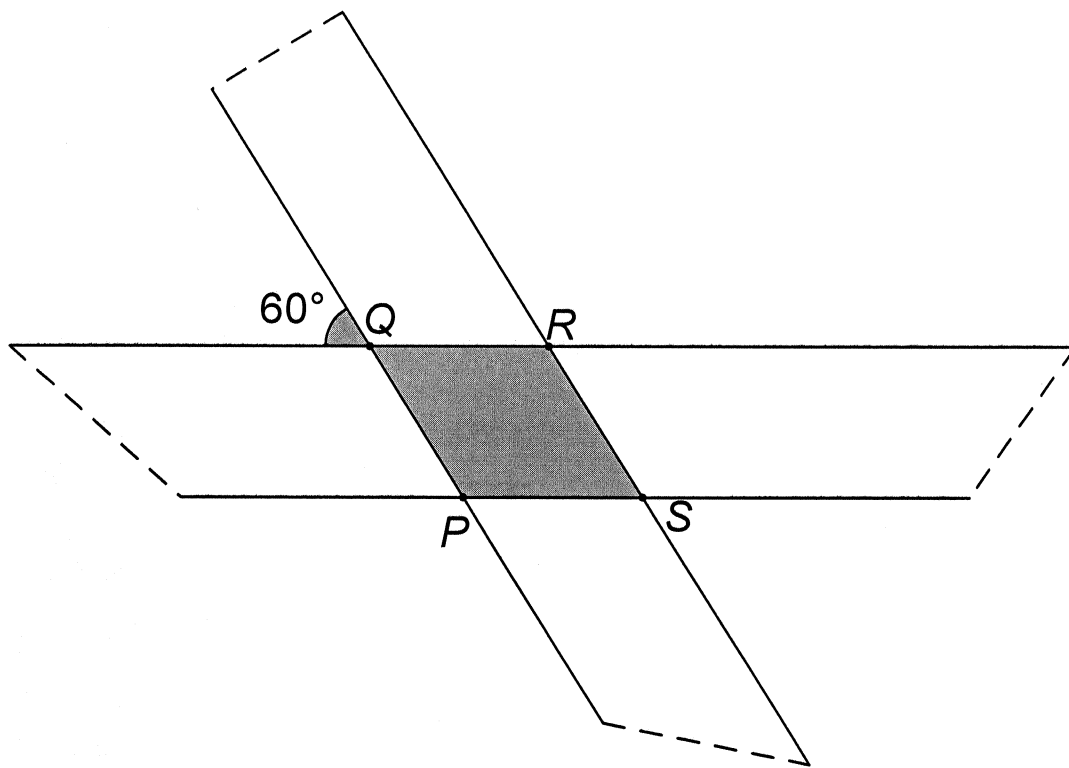**PROBLEM 5**                              **TIME:  4 minutes**

$$\frac{8\sqrt{3}}{3}$$

_____

**answer**

**Put no work on this side of the paper.  Write the answer and only the answer in the space above.  Put all work on the other side of the sheet.**

2 strips of paper 2 inches wide are crossed at a 60° angle as shown below.  Find the area of the shaded quadrilateral in square inches.

60°  Q          R

P          S

$\sin 60° = \dfrac{2}{b}$

$b = \dfrac{2}{\sin 60°} = \dfrac{2}{\frac{\sqrt{3}}{2}} = \dfrac{4}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

$A = bh = \dfrac{4\sqrt{3}}{3}(2) = \dfrac{8\sqrt{3}}{3}$

NAME: __SOLUTION__

TEAM: _____

SCHOOL: _____

**PROBLEM 6**                                    **TIME: 4 minutes**

90

_____

**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

The lengths of the sides of a triangle are consecutive integers. Half of the perimeter is 14 more than the length of the longest side. Find the perimeter.



Perimeter is $3x$

$$\frac{1}{2} \cdot 3x = 14 + x + 1$$

$$\frac{3}{2}x = x + 15$$

$$3x = 2x + 30$$

$$x = 30$$

$$3x = 90$$

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NAME: *SOLUTION*

TEAM: _____

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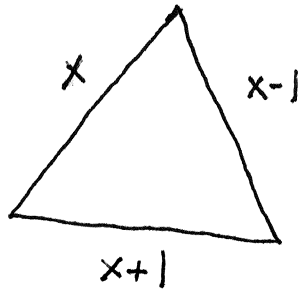
**PROBLEM 7**                                    **TIME:  4 minutes**

1:30 P.M.

_____

**answer**

**Put no work on this side of the paper.  Write the answer and only the answer in the space above.  Put all work on the other side of the sheet.**

Two children own two-way radios that have a maximum range of 3 miles. One leaves a certain point at 1:00 P.M., walking due north at a rate of 2 mi/hr. The other leaves the same point at 1:15 P.M., traveling due south at 8 mi/hr. What is the latest time at which the children will still be able to communicate with each other by radio?

$t = $ time (in hours) the northbound student walks

$t - \frac{1}{4} = $ time southbound student walks

distance = rate × time

$$2t + 8(t - \frac{1}{4}) = 3$$

$$2t + 8t - 2 = 3$$

$$10t = 5$$

$$t = \frac{1}{2} \text{ hr}$$

1:30 P.M.

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NAME: __SOLUTION__

TEAM: _____

SCHOOL: _____

**PROBLEM 8**                    **TIME: 5 minutes**

$$\frac{1}{2}$$

_____

**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

The sum of the first ten terms of an arithmetic sequence is four times the sum of the first five terms. What is the ratio of the first term to the common difference of the sequence?

For an arithmetic sequence,

$$S = \frac{n}{2}(2a + (n-1)d)$$

$$\frac{10}{2}(2a + 9d) = 4\left(\frac{5}{2}(2a + 4d)\right)$$

$$5(2a + 9d) = 10(2a + 4d)$$

$$2a + 9d = 2(2a + 4d)$$

$$2a + 9d = 4a + 8d$$

$$d = 2a$$

$$\frac{a}{d} = \frac{1}{2}$$

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NAME: ___SOLUTION___

TEAM: _____

SCHOOL: _____

**PROBLEM 9**                              **TIME: 4 minutes**

6.25

_____
**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Find the sum:

$\log_2 8 + \log_4 8 + \log_8 8 + \log_{16} 8$

$$\frac{\log_2 8}{\log_2 2} + \frac{\log_2 8}{\log_2 4} + \frac{\log_2 8}{\log_2 8} + \frac{\log_2 8}{\log_2 16}$$

$$\frac{3}{1} + \frac{3}{2} + \frac{3}{3} + \frac{3}{4}$$

$$\begin{array}{r} 3 \\ 1.5 \\ 1 \\ 0.75 \\ \hline 6.25 \end{array}$$

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NAME: _____SOLUTION_____

TEAM: _____

SCHOOL: _____

**PROBLEM 10**                                    **TIME: 5 minutes**

$p = 2\sqrt{5}$ and $-2\sqrt{5}$

_____
**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

A certain constant $p$ is chosen so that the absolute value of the difference between the roots of the equation

$$x^2 - px - p^2 = 0$$

is 10.

What are all possible values of $p$?

$$X = \frac{p \pm \sqrt{p^2 + \frac{4}{1}p^2}}{2} = \frac{p \pm \sqrt{5p^2}}{2} = p\left(\frac{1 \pm \sqrt{5}}{2}\right)$$

$$|x_1 - x_2| = |p|\left|\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right| = \sqrt{5}|p|$$

$$10 = \sqrt{5}|p|$$

$$|p| = \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

$$p = \pm 2\sqrt{5}$$